Lesson 7

Lesson 7: Magnitude

Classwork

$95 < 100$

10^2

$75,741.36$

$75,741 < 100,000$

$\boxed{10^5}$

Fact 1: *The number 10^n, for arbitrarily large positive integers n, is a big number in the sense that given a number M (no matter how big it is) there is a power of 10 that exceeds M.*

Fact 2: *The number 10^{-n}, for arbitrarily large positive integers n, is a small number in the sense that given a positive number S (no matter how small it is), there is a (negative) power of 10 that is smaller than S.*

Exercise 1

Let $M = 993,456,789,098,765$. Find the smallest power of 10 that will exceed M.

$1,000,000,000,000,000$

10^{15}

Exercise 2

Let $M = 78,491\frac{899}{987}$. Find the smallest power of 10 that will exceed M.

$78,491.91$

$78,491$ $100,000$

10^5

$10^0 = 1$
$10^1 = 10$
$10^2 = 100$
$10^3 = 1,000$
$10^{+1} = 10,000$
$10^5 = 100,000$
$10^6 = 1,000,000$

$10^0 = 1$
$10^{-1} = \frac{1}{10} = 0.1$
$10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$
$10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = .001$

Tenths   hundredths   thousandths   ten thous.

$\overline{10^3}\ \overline{10^2}\ \overline{10^1}\ \overline{10^0}\ \bullet\ \overline{10^{-1}}\ \overline{10^{-2}}\ \overline{10^{-3}}\ \overline{10^{-4}}$

Exercise 3

Let M be a positive integer. Explain how to find the smallest power of 10 that exceeds it.

3172.4836

$3172 < 10,000$

10^4

$0.01 = \dfrac{1}{100} = \dfrac{1}{10^2} = 10^{-2}$

Exercise 4

The chance of you having the same DNA as another person (other than an identical twin) is approximately 1 in 10 trillion (one trillion is a 1 followed by 12 zeros). Given the fraction, express this very small number using a negative power of 10.

$4^{-3} = \dfrac{1}{4^3}$

$\dfrac{1}{10,000,000,000,000} = \dfrac{1}{10^{13}} = 10^{-13}$
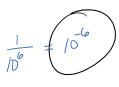
$4^{-3} = \dfrac{1}{4^3}$

Exercise 5

The chance of winning a big lottery prize is about 10^{-8}, and the chance of being struck by lightning in the U.S. in any given year is about 0.000 001. Which do you have a greater chance of experiencing? Explain.
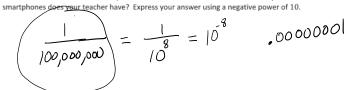
LOTTERY

10^{-8}

LIGHTNING

0.000001

10^{-6}

$\dfrac{1}{10^6} = 10^{-6}$

Exercise 6

There are about 100 million smartphones in the U.S. Your teacher has one smartphone. What share of U.S. smartphones does your teacher have? Express your answer using a negative power of 10.

$\dfrac{1}{100,000,000} = \dfrac{1}{10^8} = 10^{-8}$ $.00000001$

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Name _____   Date_____

Lesson 7: Magnitude

Exit Ticket

1. Let $M = 118,526.65902$. Find the smallest power of 10 that will exceed M.

$118,526$ $11,852,665,902$

10^6 10^{11}

2. Scott said that 0.09 was a bigger number than 0.1. Use powers of 10 to show that he is wrong.

$$\frac{9}{100} = \frac{9}{10^2} = 9 \times 10^{-2} \qquad \boxed{\frac{1}{10} = \frac{1}{10^1} = 1 \times 10^{-1}}$$

Problem Set

1. What is the smallest power of 10 that would exceed 987,654,321,098,765,432? 10^{18}

2. What is the smallest power of 10 that would exceed 999,999,999,991?

3. Which number is equivalent to 0.000 000 1: 10^7 or 10^{-7}? How do you know?

4. Sarah said that 0.000 01 is bigger than 0.001 because the first number has more digits to the right of the decimal point. Is Sarah correct? Explain your thinking using negative powers of 10 and the number line.

5. Order the following numbers from least to greatest:

$$10^5 \qquad 10^{-99} \qquad 10^{-17} \qquad 10^{14} \qquad 10^{-5} \qquad 10^{30}$$

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