Sec. 10-1 Simplifying Radical Expressions

A radical expression is an expression that contains a square root, such as $\sqrt{\frac{2GM}{R}}$. A radicand, the expression under the radical sign, is in simplest form if it contains no perfect square factors other than 1. The following property can be used to simplify square roots.

Product Property of Square Roots: For any numbers a and b, where a $\geq$ 0 and b $\geq$ 0, the square root of the product ab is equal to the product of each square root.

$\sqrt{ab} = \sqrt{a} \bullet \sqrt{b}$

$\sqrt{4 \bullet 25} = \sqrt{4} \bullet \sqrt{25}$

Ex. Simplify $\sqrt{90}$

Ex. Simplify $\sqrt{27}$

Ex. Simplify $\sqrt{150}$

ex. Simplify $\sqrt{120}$

Multiply Square Roots

ex. Simplify $\sqrt{3} \cdot \sqrt{15}$

ex. Simplify $\sqrt{5} \cdot \sqrt{10}$

ex. Simplify $\sqrt{6} \cdot \sqrt{8}$

When finding the principal square root of an expression containing variables, be sure that the result is not negative. Consider the expression $\sqrt{x^2}$. It may seem that $\sqrt{x^2} = x$. Let's look at x = -2.

$$\sqrt{x^2} = x$$

For radical expressions where the exponent of the variable inside the radical is even and the resulting simplified exponent is odd, you must use the absolute value to ensure nonnegative results.

$\sqrt{x^2} = |x|$ $\sqrt{x^4} = x^2$ $\sqrt{x^6} = |x^3|$

ex. Simplify $\sqrt{40x^4y^5z^3}$

ex. Simplify $\sqrt{32r^2s^4t^5}$

Simplify $\sqrt{56xy^{10}z^5}$

Quotient Property of Square Roots: For any numbers a and b, where a ≥ 0 and b > 0, the square root of the quotient $\frac{a}{b}$ is equal to the quotient of each square root.

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

ex. $\sqrt{\frac{49}{4}} = \frac{\sqrt{49}}{\sqrt{4}}$ $\frac{7}{2}$

If no prime factors with an exponent greater than 1 appear under the radical sign and if no radicals are left in the denominator, then a fraction containing radicals is in simplest form. Rationalizing the denominator of a radical expression is a method used to eliminate radicals from a denominator.

ex. Simplify $\sqrt{\frac{10}{3}}$

$ex. Simplify$ $\sqrt{\frac{2n}{6}}$

ex. $\dfrac{\sqrt{14}}{\sqrt{5}}$

ex. $\dfrac{\sqrt{6y}}{\sqrt{12}}$

Binomials of the form p$\sqrt{q} + r\sqrt{s}$ *and* $p\sqrt{q} - r\sqrt{s}$ are called conjugates. For example, 3 + $\sqrt{2}$ *and* $3 - \sqrt{2}$ are conjugates. Conjugates are useful when simplifying radical expressions because if p, q, r, and s are rational numbers, the product of the two conjugates is a rational number. (think about the pattern for difference of squares to find the product).

ex. Simplify 2

ex. Simplify $\frac{2}{6-\sqrt{3}}$

ex. Simplify $\frac{3}{2+\sqrt{2}}$

ex. Simplify $\frac{7}{3-\sqrt{7}}$

Simplest Radical Form: A radical expression is in simplest form when the following three conditions have been met.

1. No radicands have perfect square factors other than 1
2. No radicands contain fractions
3. No radicals appear in the denominator of a fraction

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